Where does measurement uncertainty come from?

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Quantum correlations and measurement uncertainty are inherently linked. In this context we are interested in the question whether measurement uncertainty is an inherent property of the measured quantum system or whether it is a consequence of the lack of knowledge about the measurement process. Specifically, assuming that the guessing party has access to the quantum correlations between the measurement apparatus and the system measured, is it possible for him to predict the measurement outcome with certainty? To answer these questions we use the model of the simple uncertainty games in which one performs one out of two incompatible measurements chosen uniformly at random and the uncertainty of the outcome conditional on the basis choice is evaluated [1]. In such games there will always be some uncertainty of the measurement outcome independently of the input state. Since the basis choice is probabilistic, it can be represented by a fully mixed register of dimension two. Here we consider a more complex scenario, where the "basis" register is in a coherent superposition state. We propose a measurement circuit depicted in Figure 1 where this register ρ_R can be an arbitrary qubit state. We initially also assume that the state to be measured, ρ_B, is a qubit as well. In our circuit we represent the measurement basis choice by the controlled Hadamard gate followed by a measurement in the standard basis. Hence, for the classical register the circuit reduces to probabilistically measuring σ_x or σ_z. For a coherent "basis" register, this interpretation no longer holds. However, it turns out that due to the quantum correlations that now arise between the two systems, there is a certain amount of information about the measurement outcome which becomes encoded in the "basis" register itself. We quantify the uncertainty by the probability of guessing the outcome given access to the "basis" register. We find that introducing coherence facilitates guessing and in the limit of full coherence one can achieve the guessing probability of 1 (for some specific input states ρ_B). Hence, we show that introducing a coherent register leads to a qualitatively new class of problems. Finally, we generalise our procedure to the case when the input state is of dimension N larger than two. We then replace the Hadamard in Figure 1 by its multidimensional generalisation. A natural choice for such a transformation is a Quantum Fourier Transform, because in our setup it can be used to represent the measurement in a basis which is mutually unbiased with respect to the computational one. The space of the possible outcomes is now also of dimension N, while the basis register is still just a qubit. Hence, we expect that the guessing probability of 1 is no longer achievable. We analyse this scenario by performing a numerical simulation for a 3-dimensional input state.

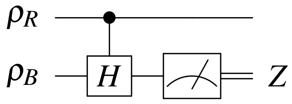

Figure 1: Quantum Circuit of the Uncertainty Game

References

[1] M. Berta, M. Christandl, R. Colbeck, J.M. Renes and R. Renner. The uncertainty principle in the presence of quantum memory, in *Nature Physics*, vol. 6, no. 9, pp. 659–662, 2010.