Entanglement distribution across a quantum peer-to-peer network

Ruben Cohen,[1,*] Emmanuel Jeandel,[2,†] Simon Perdrix,[3,‡] and Frédéric Grosshans[1,§]

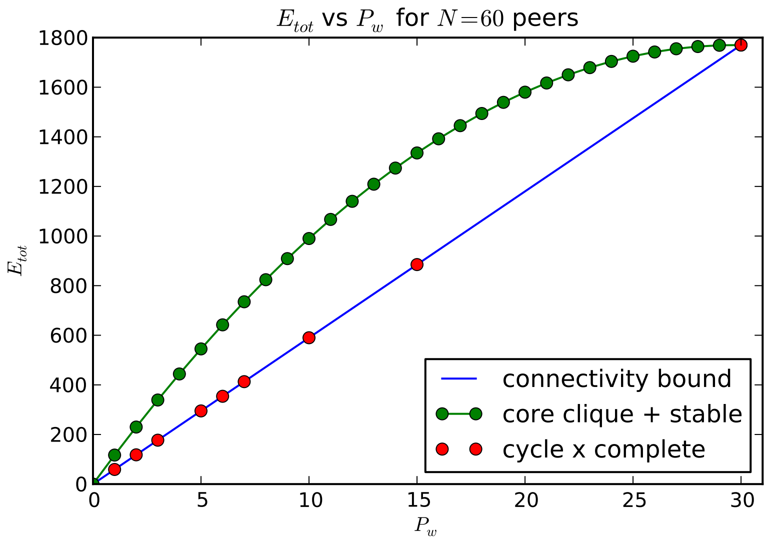
[1]*Laboratoire Aimé Cotton, CNRS, Université Paris-Sud, ENS Cachan, Orsay, France*
[2]*Université de Lorraine, Inria Project Team Carte, LORIA UMR 7503, Nancy, France*
[3]*CNRS, Inria Project Team Carte, LORIA UMR 7503, Nancy, France*

Quantum repeaters have been proposed to overcome the practical distance limit of quantum communication between two parties. Here, we go beyond this linear scheme and explore the possibilities offered by an arbitrary network of such repeaters, connecting N clients. We model these networks by undirected graphs where each vertex corresponds to a client and each edge to a quantum repeater sharing a maximally entangled pair between the two connected peers. A client can either perform a Bell measurement between two qubits or keep only one qubit; all the other qubits owned by the client are thrown away. In a single time step, two vertices (the sender and the receiver) can share entanglement if they are connected by a continuous path in the graph: the vertices along the path have to perform a Bell measurement and classically communicate their results to the sender and the receiver.

Furthermore, the above strategy allows to simultaneously share EPR pairs between several pairs of clients if the corresponding paths are vertex disjoint. If the main cost of such a network is proportional to the number of repeaters, the efficiency of a network is obtained by comparing the number P_w of EPR pairs that can be shared simultaneously to the ressources used to generate the graph *i.e.* the total number of edges E_{tot}. We have studied two figures of merit: P_w the maximum number of EPR pairs that can be shared simultaneously in the worst case, where clients are chosen by an adversary; P_a the average number of EPR pairs that can be shared simultaneously across the network when the client are chosen at random.

P_w is upper bounded by half of the minimal degree C of the graph, and hence half of the average degree: $P_w \leq \frac{C}{2} \leq \frac{E_{tot}}{N}$ and also by the non-orientable genus of the graph g: $P_w \leq g + 1$. We construct two graph families (almost) saturating this degree bound: one is the complete join of a clique of order $2P_w$ and an independent set of order $N - 2P_w$; the other is a Cartesian product of a cycle and a complete graph.

If the range l (in terms of physical distance) of a repeater is small compared to the size D of the network, it is relevant to investigate networks lying in a region of diameter D in a d-dimensional space ($d = 2$ or 3). For the average case when the client spatial density is uniform, P_a is upper bounded $\frac{P_a}{E_{tot}} = O(\frac{l}{D})$. When the range limitation l is the only connectivity limit, P_a is also lower bounded: $\frac{P_a}{E_{tot}} = \Omega[(\frac{l}{D})^d]$.



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*Electronic address: ruben.cohen1@u-psud.fr
†Electronic address: emmanuel.jeandel@loria.fr
‡Electronic address: simon.perdrix@loria.fr
§Electronic address: frederic.grosshans@u-psud.fr