Classification of spin and multipolar squeezing in spin-j collective spin systems

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Spin squeezed states in collective spin systems exhibit non-classical natures such as entanglement and quantum spin fluctuations below the standard quantum limit and they are considered to be instrumental in improving the precisions of atomic clocks and interferometers. The properties of the spin squeezing in collective spin systems of spin-$1/2$ particles are well understood [1], and spin squeezed states have been realized in ultracold and room-temperature atomic gases that provide the collective spin systems of pseudo spin-$1/2$ particles.

In such systems of pseudo spin-$1/2$ particles, observables of which quantum fluctuations can be controlled via spin squeezing are the Cartesian components of the spin vector. The multipolar observables, however, can take part in the squeezing in collective spin systems of spins higher than $1/2$, since a single spin state of a spin-j particle can be completely characterized by the $4j(j+1)$ spin and multipolar observables that generate the su($2j+1$) algebra. Not only the spin squeezing in the spin-$7/2$ atomic gas [2] but also the spin-quadrupolar squeezing has been observed in the spin-1 Bose-Einstein condensates [3]. Moreover, when we consider all available magnetic sublevels in the collective pseudo spin-$1/2$ systems, some experiments of spin squeezing can also be regarded as the spin-multipolar or multipolar squeezing [4].

In order to understand these different types of squeezing systematically, we classify the squeezing in collective spin systems of spin-j particles based on the unitary equivalence class of the sets of the squeezed and anti-squeezed observables forming the su(2) subalgebra of the su($2j+1$)

algebra. Since these observables are completely reducible representation of the su(*2*) algebra, they can be decomposed into the direct sum of the irreducible representations of the su(*2*) algebra of which dimension are not exceeding $2j+1$ and the magnitude of the structure factor f of the cyclic commutation relation is given by

$$f = \frac{j(j+1)(2j+1)}{\sum_{l=1}^{r} j_l (j_l+1)(2j_l+1)},$$

where r represents the number of the irreducible representations and j_l denotes the spin degrees of freedom of the l-th irreducible representation. This implies that the squeezings in the sets of the observables in the same unitary equivalent class give the same the squeezing limit, i.e., the minimum squeezing parameter [5]. We will also discuss the squeezing limit of the squeezed spin state generated via the one-axis twisting interaction with respect to each unitary equivalence class.

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